1. Give an example of 3 events A, B, C which are pairwise independent but not independent. Hint: find an example where whether C occurs is completely determined if we know whether A occurred and whether B occurred, but completely undetermined if we know only one of these things.
2. A bag contains one marble which is either green or blue, with equal probabilities. A green marble is put in the bag (so there are 2 marbles now), and then a random marble is taken out. The marble taken out is green. What is the probability that the remaining marble is also green?

Answer:

1. Let A, B, and C be events such that:

* P(A) = P(B) = P(C) = 1/2
* A and B are independent events
* A and C are independent events
* B and C are independent events

One possible example of such events is as follows:

* A: Tossing a fair coin results in heads
* B: Tossing a fair coin results in tails
* C: The sum of the numbers on two fair dice is even

We can see that A and B are independent events, since the outcome of one coin toss does not affect the outcome of the other. Similarly, A and C are independent events, since the sum of the numbers on the first die does not affect the sum of the numbers on the second die. Finally, B and C are independent events, since the outcome of one coin toss does not affect the sum of the numbers on two dice.

However, these events are not independent, since knowing the outcome of A and B completely determines the outcome of C. Specifically, if A and B both occur, then the sum of the numbers on the dice must be odd, and if neither A nor B occurs, then the sum of the numbers on the dice must be even. But if we know only one of A or B, then the sum of the numbers on the dice could be either even or odd.

1. Let G1 be the event that the first marble drawn is green, and G2 be the event that the second marble drawn is green. We want to find P(G2|G1), the probability that the second marble drawn is green given that the first marble drawn is green.

By Bayes' theorem, we have:

P(G2|G1) = P(G1|G2)P(G2) / P(G1)

We know that P(G2) = 1/2, since the remaining marble is either green or blue with equal probabilities. We also know that P(G1|G2) = 1, since if the second marble is green, then the first marble must also have been green.

To find P(G1), we can use the law of total probability:

P(G1) = P(G1|G2)P(G2) + P(G1|B)P(B)

where B is the event that the first marble drawn is blue. We know that P(G1|G2) = 1 and P(G2) = 1/2, as before. We also know that P(G1|B) = 0, since if the first marble is blue, then it is impossible for the second marble to be green. Therefore:

P(G1) = (1)(1/2) + (0)(1/2) = 1/2

Putting it all together, we have:

P(G2|G1) = P(G1|G2)P(G2) / P(G1) = (1)(1/2) / (1/2) = 1

Therefore, given that the first marble drawn is green, the probability that the second marble drawn is also green is 1.